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Intrinsic polarization bistability in nonlinear media

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This paper calculates the effect of a bistable polarization on the optical fields in a nonlinear medium. Bistabilities in effective refractive index and absorptivity are calculated. Bistability on reflection from a single surface is also described.

Introduction

In the talk presented at this symposium by the same authors, and published earlier in these proceedings, the way in which a nonlinear constitutive relation can lead to optical bistability was outlined. The constitutive relation for polarization in a medium was described by using the model of a Duffing oscillator, and a bistability in the polarization as a function of local electric field was derived. This paper describes the macroscopic manifestations of this microscopic bistability by calculating the effect of the bistable polarization on the optical fields. The calculation proceeds by using the nonlinear polarization derived in the earlier paper as the independent variable in the nonlinear wave equation. The optical fields are then determined from the constitutive relation. To describe observable effects, the incident wave is traced from the moment it enters a nonlinear medium. The bistable polarization creates a bistable phase relation of the light and polarization within the medium. This bistable phase relation results in bistable refractive index and absorptivity. A particularly simple method of observing these effects is in reflection from a single interface, by applying the usual boundary conditions.

Constitutive relation

The nonlinear constitutive relation that exhibits bistability was derived in the previous paper in this symposium and can be written in terms of the microscopic vibration parameter x. For simplicity we ignore local field corrections and define the macroscopic polarization P through $P = Nex$. We have

$$\{m(\omega_0^2-\omega^2)-\mathrm{i}\gamma\omega+\tfrac{3}{4}\beta|x|^2\}\,x = eE, \tag{1}$$

where ω_0 is the resonant frequency, γ is the damping and β is the coefficient of the cubic component of the nonlinear restoring force, $F = -Kx+\beta x^3$.

Nonlinear polarization

The propagation of light within the nonlinear medium is characterized by the wave equation, with the constitutive relation providing the driving term. As discussed previously, the field will be considered a function of the polarization. The wave equation therefore becomes an equation for the polarization. Under the assumption of plane-wave propagation in the z direction, we define the spatial and time dependence of the polarization by

$$P(z,t) = Ne\Gamma(z)\exp[\mathrm{i}\{\phi(z)+kz-\omega t\}]. \tag{2}$$

By using the slowly varying envelope approximation, the wave equation may be solved for $P(z,t)$, which yields

$$\ln\{\Gamma(z)/\Gamma_0\}+[\tfrac{3}{2}\beta\Delta\{\Gamma^2(z)-\Gamma_0^2\}+\tfrac{27}{64}\beta^2\{\Gamma^4(z)-\Gamma_0^4\}](\Delta^2+\gamma^2\omega^2)^{-1} = -\alpha z \tag{3}$$

and

$$\phi(z) = \phi_0+\frac{1}{2k}\left(\frac{\omega^2}{c^2}-k^2\right)z-\frac{1}{\gamma\omega}\left[\Delta\ln\{\Gamma(z)/\Gamma_0\}+\tfrac{9}{8}\beta\{\Gamma^2(z)-\Gamma_0^2\}\right], \tag{4}$$

where, for small loss, k and α satisfy the usual (linear oscillator) conditions:

$$k = \frac{\omega}{c}\left[1+\frac{\omega_\mathrm{p}^2\Delta}{\Delta^2+\gamma^2\omega^2}\right]^{\frac{1}{2}} \equiv \frac{n_0\omega}{c} \tag{5}$$

and

$$\alpha = \gamma\omega^3\omega_\mathrm{p}^2/\{2kc^2(\Delta^2+\gamma^2\omega^2)\}. \tag{6}$$

Here Γ_0 and ϕ_0 are the amplitude and phase of $P(z)$ at $z=0$ and $\omega_\mathrm{p} = (4\pi Ne^2/m)^{\frac{1}{2}}$ is the plasma frequency of the material.

Transmitted and incident flux

The equations given express the spatial and temporal dependence of the phase and amplitude of the polarization in terms of the initial conditions Γ_0 and ϕ_0, which express the amplitude and phase of the polarization at $z=0$. Because the polarization may have two stable states, this initial condition is not uniquely defined in terms of the incident field. Use of this polarization as the independent variable in the constitutive relation (1) allows us to calculate the energy flux in the nonlinear medium:

$$\bar{S} = \frac{c_\mathrm{m}^2\Gamma^2}{8\pi e^2}\left[\left(n_0+\frac{c\phi'}{\omega}\right)\{(\Delta+\tfrac{3}{4}\beta\Gamma^2)^2+\gamma^2\omega^2\}+\tfrac{3}{2}\beta\gamma c\Gamma\Gamma'\right], \tag{7}$$

where $\Gamma' = \mathrm{d}\Gamma/\mathrm{d}z$ and $\phi' = \mathrm{d}\phi/\mathrm{d}z$ are obtained from differentiation of (3) and (4).

The incident flux (which is defined for $z \leqslant 0$) that produces the flux $\bar{S}$ in the nonlinear medium is defined through boundary conditions at the incident plane ($z=0$), the boundary between a linear and a nonlinear medium. Expressing the incident flux in terms of the flux in the nonlinear medium, and then representing the latter in terms of the nonlinear polarization through (7) gives, for normal incidence and for a linear medium of refractive index n_L,

$$\bar{S}_\mathrm{I} = \left(\frac{cn_\mathrm{L}}{8\pi}\right)\left(\frac{m^2\Gamma_0^2}{16n_0^2n_\mathrm{L}^2e^2}\right)[\{\gamma\omega(n_0^2+2n_\mathrm{L}n_0+1)\}^2+\{(n_0^2+2n_\mathrm{L}n_0+1)(\Delta+\tfrac{3}{4}\beta\Gamma_0^2)+\omega_\mathrm{p}^2\}^2]. \tag{8}$$

The reflected flux is obtained in the same manner.

The independent parameter in these expressions, which describes their nonlinear behaviour, is the magnitude of the microscopic oscillation at $z=0$; i.e. Γ_0. Nonlinear transfer curves of transmitted against incident, and reflected against incident, flux are obtained by varying Γ_0.

These are shown in figure 1. The loops in the transfer curve are characteristic of the hysteresis seen in the field variables. The lower branch in transmission and the upper branch in reflection are unstable. Note that the polarization bistability, shown in the inset, does not have these loops. These curves are valid except close to the transition points, where the fluxes jump to the other branch. While the curves may not be entirely valid at these turning points, the analysis is qualitatively correct.

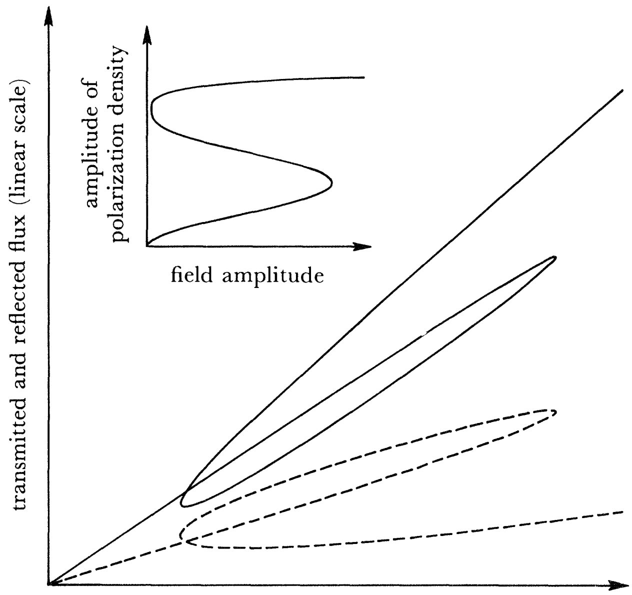

FIGURE 1. Transmitted and reflected flux (solid and broken lines, respectively), as a function of incident flux, showing bistability and hysteresis loops. Inset shows the polarization as a function of field amplitude in the nonlinear medium. This bistable curve does not show loops.

EFFECTIVE REFRACTIVE INDEX AND PHASE

The effective refractive index in the nonlinear medium may be determined in terms of the effective wavelength or by the relation between the field and the polarization. It is most convenient, however, to define the effective refractive index at $z = 0$ in terms of the ratio of the transmitted and incident flux

$$\frac{\bar{S}(0)}{\bar{S}_{\mathrm{I}}(0)} = \frac{4n_{\mathrm{eff}}\,n_2}{(n_{\mathrm{L}}+n_{\mathrm{eff}})}. \tag{9}$$

The effective index is bistable with respect to the input flux in the interesting way shown in figure 2. The arrows show the direction in which the refractive index follows the curve. The unstable branch is below either stable branch, in this case. An unusual crossing of two stable branches can be seen. In this analysis, we restrict the change in the value of the nonlinear index over which the hysteresis exists to 10%. However, by using the phenomenon of interference, this small percentage difference can be turned into a large intensity difference. This is most easily seen by studying the phase of the transmitted and reflected waves.

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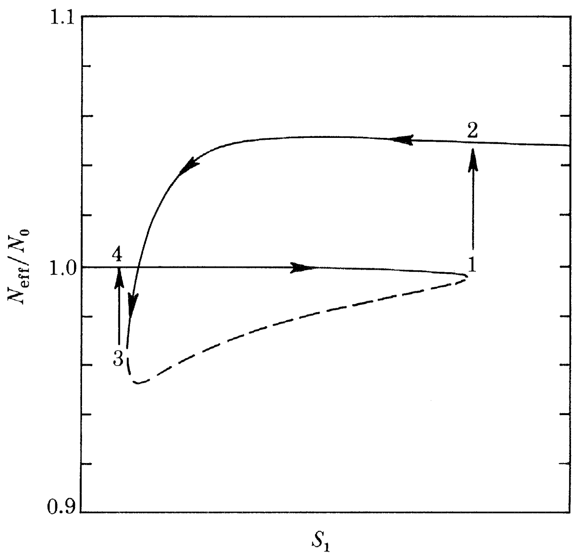

FIGURE 2. Bistable effective index of refraction n_{eff} as a function of incident flux, $\bar{S}_I$. The broken line represents the unstable portion of the curve.

The phase difference between the incident field and the polarization at $z = 0$ is also expressed in terms of the amplitude of the microscopic vibration, Γ_0:

$$\tan\phi_0 = \frac{(2n_0 n_L + n_0^2 + 1)\,\gamma\omega}{(2n_0 n_L + n_0^2 + 1)\,(\Delta + \frac{3}{4}\beta\Gamma_0^2) + \omega_p^2}. \tag{10}$$

Figure 3 shows this phase difference against incident flux, again obtained by varying Γ_0 simultaneously in ϕ_0 and $\bar{S}_I$. It can be seen that the change in the phase at switch-up and

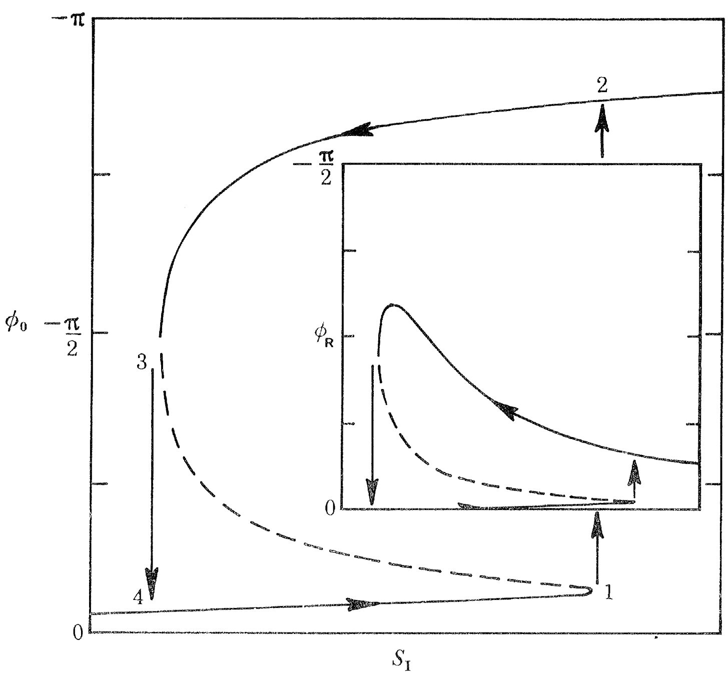

FIGURE 3. Bistable phase between the incident and transmitted field, ϕ_0 plotted as a function of the incident flux. The reflected phase ϕ_R is shown in the inset. The broken lines represent unstable solutions.

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switch-down is near π (for small loss). The inset in figure 3 shows the phase of the reflected field, also against incident flux.

This phase jump in the reflected wave can be observed as an intensity jump by arranging an interferometric experiment. This is shown in figure 4, in which a bistable reflected flux was obtained through interference with a linear reference beam. This shows that even though the nonlinearities in the amplitude variations should be small, the phase variations may be large enough to be observed.

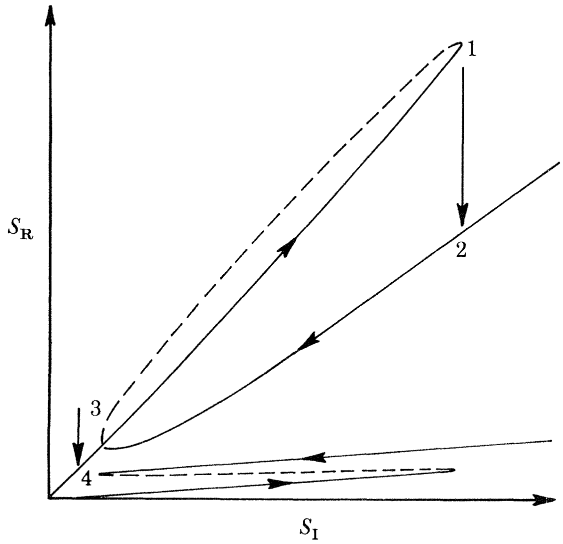

Figure 4. Bistable reflected flux obtained by interfering the reflected beam with a reference beam, plotted as a function of the incident flux.

Conclusions

We have described some of the macroscopic manifestations of microscopic bistability due to one form of the nonlinear constitutive relation. These include bistability in the refractive index and absorptivity, as well as in the intensity and phase of transmitted and reflected waves at a linear–nonlinear interface. It should be stressed that the interface is only an artifice to keep track of incident and transmitted beams and is not fundamental to the observation of bistability. Bistability can be observed at all angles of incidence to the interface; in fact, the angle of refraction itself is bistable. These characteristics set this effect apart from previous nonlinear interface effects (Smith *et al.* 1979).

As discussed by Garmire, Poole & Goldstone (this symposium), observation of bistability of this nature will require large nonlinearities and narrow lines (small damping constants). Perhaps the first place to look for such bistability is in the microwave region.

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Reference

Smith, P. W., Hermann, J.-P., Tomlinson, W. J. & Moloney, P. J. 1979 *Appl. Phys. Lett.* **35**, 846–848.